Staggered Domain Wall Fermions

George T. Fleming [a] and Pavlos M. Vranas [b]

[a] Physics Department, The Ohio State University, Columbus, OH 43210-1168, USA.

[b] IBM T.J. Watson Research Center, Route 134, Yorktown Heights, NY 10598, USA.

Staggered Domain Wall Fermions (SDWF) combine the attractive chiral properties of staggered fermions with those of domain wall fermions. SDWF describe four flavors with exact U(1)×U(1) flavor chiral symmetry. An extra lattice dimension is introduced and the full SU(4)×SU(4) flavor chiral symmetry is recovered as its size is increased. Here, the free theory of SDWF is described and a preliminary discussion of the interacting case is presented. SDWF may be well suited for numerical simulation of lattice QCD thermodynamics.

1. Introduction

Numerical simulations of QCD thermodynamics, using current methods and supercomputers do not realize the flavor chiral symmetries as well as one would like. More specifically the study of the relevant critical phenomena close to the finite temperature transition may be affected. It is reasonable to investigate alternatives to current methods especially if they combine some of their attractive features.

Staggered fermions, and their variations, describe four flavors (in four dimensions) and have an exact U(1)×U(1) flavor chiral symmetry. The full SU(4)×SU(4) symmetry is recovered when the lattice spacing a approaches zero. Two flavor simulations of QCD are done by taking the "square root" of the staggered determinant. Because the U(1)×U(1) chiral symmetry is exact at any a staggered fermions have been attractive for studies of spontaneous chiral symmetry breaking phenomena. On the other hand only one pion in the multiplet acts as a Goldstone boson due to the flavor breaking. Even the highly improved `asqtad` action may require $a^{-1} \gtrsim 8T_c$ to get all pions lighter than the kaon mass [1].

A few years ago new methods were developed with good chiral symmetry properties: please see the review articles in [2, 3, 4, 5, 6, 7, 8, 9] and references therein. One of them, domain wall fermions (DWF), uses standard Wilson fermions but introduces an extra dimension with L_s sites and free boundary conditions. As a result massless surface modes develop, with one chirality bound exponentially on one wall and the other on the other wall. At finite L_s chiral symmetry is broken by their overlap but it is restored exponentially fast as L_s is increased. At $L_s = \infty$ (the overlap formalism [2, 6]) the chiral symmetry is exact, even at non zero a and topological zero mode effects are reproduced. These remarkable properties make DWF a good candidate for QCD thermodynamic studies. For such studies see [10, 11, 12, 13].

Both staggered and DWF are very close to realistic simulations of QCD thermodynamics. The present work is an attempt to combine their attractive features. SDWF are staggered fermions defined in space time with one extra direction and free boundary conditions. As with DWF, surface modes develop with the plus chirality of all flavors localized on one boundary (wall) and the other chirality on the other boundary. By properly identifying the various flavors on the boundaries one can construct four flavors of Dirac spinors with exact U(1)×U(1) at non-zero a and finite L_s. The full SU(4)×SU(4) symmetry is recovered in the $L_s \to \infty$ limit even for non zero a.

Here, the free theory of SDWF is described and a preliminary discussion of the interacting case is presented. At this moment it is not clear to what degree the localization properties persist in the strong interaction regime where lattice QCD thermodynamic simulations are presently done.

2. Staggered Domain Wall Fermions

We start our construction of the free SDWF action by writing the free staggered action in momentum space and in the hypercubic notation of Kluberg-Stern *et al.* [14]

$$S_{\rm sf} = \sum_k \overline{Q}(k)\left[m(\mathbb{1}\otimes\mathbb{1})+ \not{D}(k)\right]Q(k) \qquad (1)$$

$$\not{D}(k) = \sum_\mu \left[\tfrac{i}{2}\sin k_\mu(\gamma_\mu\otimes\mathbb{1}) + b_\mu(\gamma_5\otimes\xi_{5\mu})\right] \qquad (2)$$

with $b_\mu = (1-\cos k_\mu)/2$. The lattice coordinates x_μ are decomposed into the coordinates of 2^d hypercubes y_μ and coordinates of sites within a given hypercube $A_\mu = (x_\mu \bmod 2) \in \{0,1\}$ so that $x_\mu = 2y_\mu + A_\mu$. The momenta k_μ are reciprocal to the hypercubes y_μ.

To complete the construction of the free SDWF action, we introduce an extra dimension in the s direction with free boundaries, sum $S_{\rm sf}$ over the s-slices and add terms to the action that project on $(\gamma_5\otimes\mathbb{1})$ chiralities

$$S_{\rm sdwf} = \sum_s \left[S_{\rm sf} + \sum_k \overline{Q}(k,s)D_5 Q(k,s)\right] \qquad (3)$$

$$D_5(s,s') = \tfrac{1}{a_5}\left[P_+\delta_{s+1,s'} + P_-\delta_{s-1,s'} - \delta_{s,s'}\right]. \qquad (4)$$

Explicitly, the chiral projectors are $P_\pm \equiv ((\mathbb{1}\pm\gamma_5)\otimes\mathbb{1})/2$. Even though D_5 breaks the U(1)×U(1) symmetry generated by $(\gamma_5\otimes\xi_5)$, we will show in section 3 that an extended symmetry exists to protect mass terms like $m(\mathbb{1}\otimes\mathbb{1})\delta_{s,s'}$ from additive renormalizations. Thus, we find it natural to set $m\to\frac{1}{a_5}$ and drop these terms from the action. However, it is important to keep in mind that at higher energy scales, fermions propagate like *massive* staggered fermions in d dimensions.

Fully chiral flavor symmetric modes should exist at lower energy scales in the SDWF theory if we can find normalizable states that satisfy the following zero mode equation

$$\begin{aligned}\sum_{s'} \Big\{ &\tfrac{1}{a_5}\left[P_+\delta_{s+1,s'} + P_-\delta_{s-1,s'}\right] \\ + \quad &\sum_\mu(\gamma_5\otimes\xi_{5\mu})b_\mu\delta_{s,s'}\Big\}\,\Phi(k,s') = 0. \end{aligned} \qquad (5)$$

From this equation, it is easy to see that the $P_\pm$ projectors in the s-dependent part should commute with the flavor breaking part so that each may be simultaneously diagonalized. This constraint alone effectively restricts the allowed projectors to the ones we have chosen. We define an additional set of operators to project out specific flavor components as well: $P_{+\pm} \equiv (P_+\otimes(\mathbb{1}\pm\xi_5))/2$. The solution is separable and $\phi(s)$ is the s-dependent part. We write it block notation using the new projectors:

$$\phi(s)^T = \left(\phi_{++}^T, \phi_{+-}^T, \phi_{-+}^T, \phi_{--}^T\right) \qquad (6)$$

where $\phi_{-+}(s) = P_{-+}\phi(s)$, etc. In this notation, we can write

$$\sum_\mu(\gamma_5\otimes\xi_{5\mu})b_\mu = \begin{pmatrix} & B & & \\ -B^\dagger & & & \\ & & & -B \\ & & B^\dagger & \end{pmatrix}. \qquad (7)$$

The solutions to (5) (after iterating one time) are

$$\begin{aligned}\phi_{\pm+}(s\pm2) &= -a_5^2 BB^\dagger\phi_{\pm+}(s)\\ \phi_{\pm-}(s\pm2) &= -a_5^2 B^\dagger B\phi_{\pm-}(s).\end{aligned} \qquad (8)$$

Solving the equations relating nearest neighbor s sites is more complicated because the flavor components mix and will be discussed elsewhere [15].

For free fermions, $[B,B^\dagger]=0$ and $a_5^2BB^\dagger$, $a_5^2B^\dagger B$ are both proportional to the identity with eigenvalue

$$\lambda(a_5^2BB^\dagger) = \lambda(a_5^2B^\dagger B) = a_5^2\sum_\mu b_\mu^2. \qquad (9)$$

If we require that $a_5^2\sum_\mu b_\mu^2 < 1$ then for a semi-infinite s direction, $s\geq0$, only $\phi_{+\pm}$ is normalizable, while $\phi_{-\pm}$ is not. However, this is not enough to ensure that the doubler modes are not present. One must further require that the above condition excludes momenta with components larger or equal to π. This is satisfied provided that $a_5\geq1$. If one wanted to further restrict the momenta to $\pi/2$ then the requirement is that $a_5\geq2$.

Most of the recent work in the overlap formalism is related to taking the $a_5\to0$ limit. Clearly, additional terms must be added to our SDWF action to maintain the cutoff of doubler momenta as $a_5\to0$. Possible terms are currently under study. In any construction of a staggered overlap Hamiltonian, it will be important to demonstrate that naive fermions are not recovered in the chiral limit and that the U(1)×U(1) symmetry remains exact. Of course, it is always possible to use the overlap formalism directly with finite a_5.

When adding interactions to a free staggered action, it is usually a good idea to first transcribe the free action in terms of single component per site fields in position space. The unitarily equivalent hypercubic basis of Daniel and Sheard [16] is ideally suited for this purpose. In this basis, fermion bilinears are written $\overline{\chi}_A(y)(\overline{\gamma_S \otimes \xi_F})_{AB}\chi_B(y)$, where the fermion fields $\chi_A(k)$ are directly the single component fields on the corners of the hypercube y: $\chi(x) = \chi(2y+A) = \chi_A(y)$. The components of the spin-flavor matrices are given by

$$\left(\overline{\gamma_S \otimes \xi_F}\right)_{AB} = \frac{1}{2^{d/2}}\mathrm{tr}[\gamma_A^\dagger \gamma_S \gamma_B \gamma_F^\dagger] \in \{0, \pm 1\} \quad (10)$$

where S and F are d-dimensional binary vectors like A and B and $\gamma_S \equiv \gamma_1^{S_1} \times \cdots \times \gamma_d^{S_d}$.

The staggered $\not{D}$ is the usual one

$$\not{D} = \sum_\mu (-1)^{\eta_\mu(x)} \left[\delta_{x+\hat{\mu},x'} - \delta_{x,x'+\hat{\mu}}\right] \quad (11)$$

where $\eta_\mu(x) = \sum_{\nu<\mu} x_\nu$. For the projection terms in D_5 proportional to $(\overline{\gamma_5 \otimes 1\!\!1})$, we have

$$\begin{aligned} &\overline{\chi}(y,s)(\overline{\gamma_5 \otimes 1\!\!1})\chi(y,s\pm 1) \\ &\quad = \quad (-1)^{\varphi(x)}\overline{\chi}(x,s)\chi(\overline{x},s\pm 1) \end{aligned} \quad (12)$$

where $\varphi(x) = d/2 + \sum_{\mu=1}^{d/2} x_{2\mu-1}$ and $\overline{x}$ is the opposite corner of the hypercube: $\overline{x}_\mu = x_\mu + 1 - 2(x_\mu \bmod 2)\ \forall\ \mu$.

3. Symmetries

When constructing the SDWF action, it is important to preserve all the symmetries of the massless staggered action [17]. Of course, adding any new terms to the staggered action will likely break some of those symmetries, so we have to find new symmetries that involve the extra dimension. However, we will still have to show that these new symmetries in the space with an extra direction are maintained by the action that has all fields integrated out except the "light" ones at the boundaries. We present here the symmetry transformations in the hypercubic notation for the space with an extra dimension.

Rotations by $\pi/2$. These rotations are in planes perpendicular to the extra dimension and the transformations are the same as the original staggered ones.

μ-parity. These transformations reflect the $d-1$ spatial axes perpendicular to the spatial axis in the $\hat{\mu}$ direction. D_5 is not invariant under this symmetry unless we also reflect the s direction as well. If we define the reflection operator $\mathcal{R}_{s,s'} \equiv \delta_{L_s-1-s,s'}$, the transformation is

$$\begin{aligned} Q(y,s) &\rightarrow (\gamma_\mu \otimes \xi_5)\mathcal{R}_{s,s'}\ Q(y,s'), \\ \overline{Q}(y,s) &\rightarrow \overline{Q}(y,s')\ \mathcal{R}_{s',s}(\gamma_\mu \otimes \xi_5). \end{aligned} \quad (13)$$

Shift by one lattice spacing. By inspection, one can see that (11) is invariant under shifts by one lattice spacing in any spatial direction μ. Because of the additional structure imposed on the lattice by the hypercubic formulation, the symmetry transformation, while still valid, is complicated. For SDWF, there is an added complication that some parts of the transformation require a reflection in the s direction (with s indices suppressed):

$$\begin{aligned} Q(y) \rightarrow\ & \tfrac{1}{2}\left[(1\!\!1 \otimes \xi_\mu) - (\gamma_{\mu 5} \otimes \xi_5)\mathcal{R}\right] Q(y) \\ & + \tfrac{1}{2}\left[(1\!\!1 \otimes \xi_\mu) + (\gamma_{\mu 5} \otimes \xi_5)\mathcal{R}\right] Q(y+\hat{\mu}), \\ \overline{Q}(y) \rightarrow\ & \overline{Q}(y)\tfrac{1}{2}\left[(1\!\!1 \otimes \xi_\mu) - \mathcal{R}(\gamma_{5\mu} \otimes \xi_5)\right] \\ & + \overline{Q}(y+\hat{\mu})\tfrac{1}{2}\left[(1\!\!1 \otimes \xi_\mu) + \mathcal{R}(\gamma_{5\mu} \otimes \xi_5)\right]. \end{aligned} \quad (14)$$

$\mathrm{U}(1)_e \times \mathrm{U}(1)_o$ *chiral rotations.* The residual chiral symmetry of staggered fermions involves making separate chiral rotations on even and odd sites. Terms in D_5 are not invariant under these rotations unless we extend the notion of even and odd, *including the extra dimension.* First, we define the operator $\mathcal{S}_{s,s'} \equiv (-1)^s \delta_{s,s'}$ and then the extended even/odd projection operators

$$\begin{aligned} P_e &= \tfrac{1}{2}\left[(1\!\!1 \otimes 1\!\!1) + \mathcal{S}(\gamma_5 \otimes \xi_5)\right], \\ P_o &= \tfrac{1}{2}\left[(1\!\!1 \otimes 1\!\!1) - \mathcal{S}(\gamma_5 \otimes \xi_5)\right]. \end{aligned} \quad (15)$$

Using these projection operators the chiral transformation is

$$\begin{aligned} Q(y) &\rightarrow \left(e^{i\theta_e}P_e + e^{i\theta_o}P_o\right)\ Q(y), \\ \overline{Q}(y) &\rightarrow \overline{Q}(y)\ \left(e^{-i\theta_o}P_e + e^{i\theta_e}P_o\right). \end{aligned} \quad (16)$$

4. Flavors of SDWF

From section 2 one sees that for a finite extra direction with L_s sites the P_+ components of all flavors are localized around $s{=}0$ while the P_- components are localized around $s{=}L_s{-}1$. From section 3, however, we mentioned the importance of constructing an effective d-dimensional field q from the surface states consistent with all the SDWF symmetries, particularly the $\mathrm{U}(1)_e{\times}\mathrm{U}(1)_o$ chiral symmetry. For example, to project flavor components with P_{++}, one should choose s near zero. If $s{=}0$ is chosen, $P_{++}q(y) = P_{++}Q(y,0)$, then these components also belong to the P_e part of the fermion field. Therefore, to project flavor components with P_{-+} one is not only restricted to choose s near $L_s{-}1$ but also choose s so these components belong to the P_o part of the fermion field. Then, components $P_{\pm+}q$ will not mix even for finite L_s because of the even/odd symmetry. In this example, one would like to pick $P_{-+}q(y) = P_{-+}Q(y,s)$ with s being even and near $L_s{-}1$. So, if L_s is odd then $s{=}L_s{-}1$ is a good choice. However, if L_s is even, then one should choose $s{=}L_s{-}2$ instead.

$$
\underset{Q}{\begin{bmatrix} \begin{pmatrix} \mathrm{X} \\ x \\ x \\ x \end{pmatrix} \\ \begin{pmatrix} x \\ \mathrm{X} \\ x \\ x \end{pmatrix} \\ \vdots \\ \begin{pmatrix} x \\ x \\ \mathrm{X} \\ x \end{pmatrix} \\ \begin{pmatrix} x \\ x \\ x \\ \mathrm{X} \end{pmatrix} \end{bmatrix}}
=
\underset{P_eQ}{\begin{bmatrix} \begin{pmatrix} \mathrm{X} \\ 0 \\ 0 \\ x \end{pmatrix} \\ \begin{pmatrix} 0 \\ \mathrm{X} \\ x \\ 0 \end{pmatrix} \\ \vdots \\ \begin{pmatrix} x \\ 0 \\ 0 \\ x \end{pmatrix} \\ \begin{pmatrix} 0 \\ x \\ x \\ 0 \end{pmatrix} \end{bmatrix}}
+
\underset{P_oQ}{\begin{bmatrix} \begin{pmatrix} 0 \\ x \\ x \\ 0 \end{pmatrix} \\ \begin{pmatrix} x \\ 0 \\ 0 \\ x \end{pmatrix} \\ \vdots \\ \begin{pmatrix} 0 \\ x \\ \mathrm{X} \\ 0 \end{pmatrix} \\ \begin{pmatrix} x \\ 0 \\ 0 \\ \mathrm{X} \end{pmatrix} \end{bmatrix}}
\tag{17}
$$

Using the block notation of (6), an example for even L_s is sketched in (17). In this equation $Q(s{=}0)$ is at the top and $Q(s{=}L_s{-}1)$ is at the bottom. The capital letters denote one of the "correct" choices. On the other hand if L_s is odd, say 3 then

$$
\underset{Q}{\begin{bmatrix} \begin{pmatrix} \mathrm{X} \\ \mathrm{X} \\ x \\ x \end{pmatrix} \\ \begin{pmatrix} x \\ x \\ x \\ x \end{pmatrix} \\ \begin{pmatrix} x \\ x \\ \mathrm{X} \\ \mathrm{X} \end{pmatrix} \end{bmatrix}}
=
\underset{P_eQ}{\begin{bmatrix} \begin{pmatrix} \mathrm{X} \\ 0 \\ 0 \\ x \end{pmatrix} \\ \begin{pmatrix} 0 \\ x \\ x \\ 0 \end{pmatrix} \\ \begin{pmatrix} x \\ 0 \\ 0 \\ \mathrm{X} \end{pmatrix} \end{bmatrix}}
+
\underset{P_oQ}{\begin{bmatrix} \begin{pmatrix} 0 \\ \mathrm{X} \\ x \\ 0 \end{pmatrix} \\ \begin{pmatrix} x \\ 0 \\ 0 \\ x \end{pmatrix} \\ \begin{pmatrix} 0 \\ x \\ \mathrm{X} \\ 0 \end{pmatrix} \end{bmatrix}}
\tag{18}
$$

We note that other choices for selecting flavor components near the boundaries are certainly possible. We also note that the even L_s choice in (17) does not transform simply under the $\mathcal{R}$ reflections of section 3 and requires modification to make it consistent with the other staggered symmetries. This will be discussed in more detail elsewhere [15].

5. The SDWF propagator

The SDWF propagator for the Dirac matrix given in section 2 is presented here in a general form. The detailed form will be presented elsewhere [15]. A degenerate mass term that explicitly mixes chiralities is added. For odd L_s it has the form

$$
\begin{aligned}
M_{s,s'} = &-im_f \delta_{s,L_s-1}\delta_{0,s'}P_+ \\
&+im_f \delta_{s,0}\delta_{L_s-1,s'}P_- .
\end{aligned}
\tag{19}
$$

For even L_s it has a form according to the discussion in section 4. The propagator in momentum space has the general form:

$$
\begin{aligned}
&D^{-1}(s,s') = \\
&\quad [G_1 + m_f G_2]\epsilon(s-s') + G_3\epsilon(s-s'-1)
\end{aligned}
\tag{20}
$$

where G_1, G_2 and G_3 are functions of momenta and are proportional to the identity in their flavor indices. Also, G_1 anti-commutes with $(\gamma_5 \otimes \mathbb{1})$ while G_2 and G_3 commute with $(\gamma_5 \otimes \mathbb{1})$. The flavor mixing is in the function ϵ:

$$\begin{array}{rclr} \epsilon(x) & = & (\mathbb{1} \otimes \mathbb{1}), & (x \text{ even}) \\ \epsilon(x) & = & \dfrac{\sum_\mu (\mathbb{1} \otimes \xi_{5\mu}) b_\mu}{|b|}, & (x \text{ odd}) \end{array} \quad (21)$$

where $|b| = \sqrt{\sum_\mu b_\mu^2}$. For $s - s'$ even and m_f=0 the propagator anti-commutes with $(\gamma_5 \otimes \mathbb{1})$ and has no flavor mixing except for the last term in (20). In this term ϵ(odd) breaks flavor in exactly the same way as free staggered fermions. An exact U(1)×U(1) symmetry is maintained. The matrix coefficient G_3 vanishes exponentially fast with L_s for s, s' near opposing boundaries and therefore as $L_s \to \infty$ with m_f=0 the propagator anti-commutes with $(\gamma_5 \otimes \mathbb{1})$ and has no flavor mixing provided $s - s'$ is even. This is in accordance with the discussion in section 4.

Finally, if we add to b_μ a constant term $1/a_5 - m_0$: $b_\mu = (1 - \cos k_\mu + 1/a_5 - m_0)/2$, the effective mass m_{eff} has exactly the same form as in Wilson DWF but the decay coefficient is now in terms of $|b|$ instead of $b = \sum_\mu [1 - cos(p_\mu)] + 1/a_5 - m_0$. In this case, the localization condition for m_0 is the same as in DWF and the $a_5 \to 0$ limit can be taken the same way as in DWF. Furthermore, this term may be needed to cancel any renormalization of the flavor breaking term. This term would appear in the action as $(1/a_5 - m_0)/2 \sum_\mu \overline{Q}(\gamma_5 \otimes \xi_{5\mu})Q$, a term that is not invariant under shifts by a single lattice spacing. However, such a term, or a generalization of it, might be needed for the above reasons.

6. The SDWF transfer matrix

We can use the technique of Neuberger [18] to rewrite the free SDWF determinant in a form that allows us to quickly identify the transfer matrix. We use the same basis as in (7). After interchanging various rows and columns of the SDWF matrix, the determinant is equivalent to the determinant of the matrix

$$\begin{pmatrix} \alpha_0 & & & \beta_0 \\ \beta_1 & \ddots & & \\ & \ddots & \alpha_{L_s-2} & \\ & & \beta_{L_s-1} & \alpha_{L_s-1} \end{pmatrix} \quad (22)$$

where all of the α_s and β_s are the block triangular matrices

$$\alpha_s = \begin{pmatrix} \mathcal{B} & 0 \\ -\mathcal{C} & 1/a_5 \end{pmatrix},\ \beta_s = \begin{pmatrix} 1/a_5 & \mathcal{C}^\dagger \\ 0 & -\mathcal{B} \end{pmatrix}. \quad (23)$$

For α_{L_s-1} and β_0, $1/a_5$ is replaced with $-\mu/a_5$ so μ is a parameter that controls the boundary conditions: $\mu = \pm 1$ for (anti)periodic and $\mu = 0$ for free. The definition of $\mathcal{B}$ follows from (7)

$$\mathcal{B} = \begin{pmatrix} 0 & B \\ -B^\dagger & 0 \end{pmatrix} \quad (24)$$

and the definition of $\mathcal{C}$ follows from (2)

$$\sum_\mu \frac{i}{2} \sin k_\mu (\gamma_\mu \otimes \mathbb{1}) = \begin{pmatrix} 0 & -\mathcal{C} \\ \mathcal{C}^\dagger & 0 \end{pmatrix} \quad (25)$$

In this notation, following Neuberger's construction leads to the free SDWF transfer matrix

$$T = \begin{pmatrix} \mathcal{B}^{-1}/a_5 & -\mathcal{B}^{-1}\mathcal{C} \\ -\mathcal{C}^\dagger \mathcal{B}^{-1} & a_5(\mathcal{C}^\dagger \mathcal{B}^{-1} \mathcal{C} - \mathcal{B}) \end{pmatrix}. \quad (26)$$

Since we chose to set the staggered mass $m \to \frac{1}{a_5}$ in (3), then $\mathcal{B}$ is strictly anti-Hermitian, so T is as well (This is different from Wilson DWF and gives some idea why solving the zero mode problem in (8) simplifies when solving for the field two sites away). In this case taking the $a_5 \to 0$ limit in order to identify the Hamiltonian does not make sense since the doublers are re-introduced (see section 2). On the other hand if we do not set the staggered mass $m \to \frac{1}{a_5}$ then the $a_5 \to 0$ limit does not re-introduce the doublers and can be taken. However this term breaks the exact U(1)×U(1) symmetry. An alternative is to add an $\frac{1}{a_5} - m_0$ term as in section 5. This does not break the U(1)×U(1) symmetry, allows the $a_5 \to 0$ limit to be taken without reintroducing the doublers, but breaks the shift by one lattice spacing symmetry.

7. Alternative actions

The SDWF action considered here is not unique. Better actions may be constructed using improved fields in the same spirit as with staggered fermions (see [19, 20] and references therein). But even in the spin/flavor basis considered here one could add the domain wall part in slightly different ways. For example one could have added to the standard staggered action the exact same term as the one added in Wilson DWF but multiplied by $\sum_\mu(\gamma_5 \otimes \xi_{5\mu})$.

8. Conclusions

Staggered domain wall fermions (SDWF) have been constructed for the free theory. They describe four flavors with exact U(1)×U(1) flavor chiral symmetry. The full SU(4)×SU(4) flavor chiral symmetry is recovered as the size of the extra dimension is increased. The addition of interactions for QCD is straight forward but it remains to be seen how well the SDWF properties are maintained at the stronger couplings where numerical simulations are done. It is hoped that SDWF may be useful for numerical simulations of QCD thermodynamics.

REFERENCES

1. U. M. Heller *et al.*, these proceedings, hep-lat/0110067.
2. R. Narayanan and H. Neuberger, Nucl. Phys. Proc. Suppl. **34**, 587 (1994) [hep-lat/9311015].
3. M. Creutz, Nucl. Phys. Proc. Suppl. **42**, 56 (1995) [hep-lat/9411033].
4. Y. Shamir, Nucl. Phys. Proc. Suppl. **47**, 212 (1996) [hep-lat/9509023].
5. T. Blum, Nucl. Phys. Proc. Suppl. **73**, 167 (1999) [hep-lat/9810017].
6. H. Neuberger, Nucl. Phys. Proc. Suppl. **83**, 67 (2000) [hep-lat/9909042].
7. P. M. Vranas, Nucl. Phys. Proc. Suppl. **94**, 177 (2001) [hep-lat/0011066].
8. R. G. Edwards, these proceedings.
9. Y. Kikukawa, these proceedings.
10. G. T. Fleming, Nucl. Phys. A **663**, 979 (2000) [hep-ph/9910453].
11. P. M. Vranas, Nucl. Phys. Proc. Suppl. **83**, 414 (2000) [hep-lat/9911002].
12. P. Chen *et al.*, Phys. Rev. D **64**, 014503 (2001) [hep-lat/0006010].
13. G. T. Fleming, Nucl. Phys. Proc. Suppl. **94**, 393 (2001) [hep-lat/0011069].
14. H. Kluberg-Stern, A. Morel, O. Napoly and B. Petersson, Nucl. Phys. B **220**, 447 (1983).
15. G. Fleming and P.Vranas in preparation.
16. D. Daniel and S. N. Sheard, Nucl. Phys. B **302**, 471 (1988).
17. T. Jolicoeur, A. Morel and B. Petersson, Nucl. Phys. **B274**, 225 (1986).
18. H. Neuberger, Phys. Rev. **D57**, 5417 (1998) [hep-lat/9710089].
19. Y. Luo, Phys. Rev. D **55**, 353 (1997) [hep-lat/9604025].
20. W. J. Lee and S. R. Sharpe, Phys. Rev. D **60**, 114503 (1999) [hep-lat/9905023].